PREFACE

by Jerry Choe

This is the exerpt from from Betz's original manuscript, where he dismisses Drag as a "loss" and says that wings create minimum velocity loss from drag and therefore we must use wings. However, based on Newton's Law of Conservation of Energy, energy cannot be lost. It can only be transformed. Therefore Betz's reasoning for using Lift over Drag is flawed. Drag can capture far more energy than Lift.

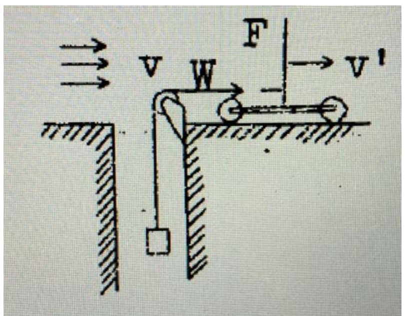

Fig.1

EXERPT FROM BETZ:

"[T]he force W exerted by the air on the object involved a loss of energy, in that the work W(v - v 1) was converted into vortical energy. This leads us to inquire as to whether work is conceivable without this loss of energy. This is actually the case. If a suitably shaped object is chosen, the power P does not lie in the direction of the relative air flow, but forms an acute angle with it (Fig.2). If this force is resolved into a component W in the direction of the air flow and a component A perpendicular to it, then only the component W, <u>the real resistance or "drag" determines the amount of energy consumed in the production of vortices.</u> The energy lost per second (L1ost) is Wc, in which c is the wind velocity relative to the object . The other component, the "lift" causes no loss.of energy, since it is perpendicular to the direction of motion. This lifting force is theref ore similar to a centripetal force, since it only deflects the air from its course without affecting it energy. In order to obtain great efficiency with only a small loss of energy, objects must be used of such a shape that they will yield the greatest possible lift and the smallest possible drag. Such objects are called "wings".

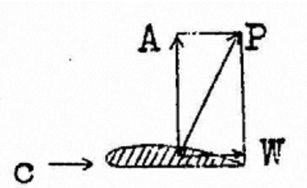

Fig.2 Forces acting on a wing.

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WINDMILLS IN THE LIGHT OF MODERN RESEARCH

By A. Betz

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TECHNICAL MEMORANDUM NO. 474.

WINDMILLS IN THE LIGHT OF MODERN RESEARCH.*

By A. Betz.

I. Introduction

Windmills have received much more attention since the war. The increased interest is mainly due to two causes: first, the temporary coal shortage which intensified the search for new sources of energy; and secondly, the impression that the lessons learned in aeronautics would produce a considerable improvement in windmills. In fact, the general progress in aerodynamics has had a beneficial effect on windmill construction. No remarkable improvement in their efficiency has, however, been attained and can hardly be expected. This, again, confirms an often observed fact, that technical devices which have passed through a long period of development, have frequently attained, simply through experience, such a high degree of perfection that modern scientific knowledge can make only relatively slight additional improvements in them. The chief contribution of modern research in the field of windmills is a better understanding of the phenomena and of the available means for the accomplishment of certain results, but also of the natural limits to their productive capacity.

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*"Die Windmühlen im Lichte neuerer Forschung," Die Naturwissenschaften, Vol. XV, No. 46, November 18, 1927, pp. 905-914.

The task of a windmill is to extract energy from the wind and convert it into an available useful form. Water turbines have a similar task, but there is one very decided difference. In water-power plants there is generally but a very limited amount of energy available, which is extracted by relatively expensive power plants. When the energy in the turbine is converted into a useful form, it is very important for it to be done with the maximum efficiency, since every loss in the conversion process means a corresponding waste of the costly energy. The large capital investment and the interest on it also play an important role. With windmills we have at hand such immense quantities of energy in the great ocean of air, that we cannot possibly use it all. It is therefore relatively unimportant as to how much energy is lost in the process of transformation, the cost being the only important thing. The most economical windmill is the one which furnishes the kilowatt-hour at the lowest figure. Since, in the production of energy with a windmill, the principal part of the cost is the interest and amortization of the original investment, the problem is chiefly to obtain the maximum output with the minimum investment, i.e., to make the ratio of the investment to the annual output as small as possible.

In the production of energy by windmills, this main problem is rendered still more difficult by other problems, due partly to the character of the wind and partly to the particular purpose to be served. The most serious difficulty is the irregularity of the wind. During a large part of the time the wind is too weak to turn the windmill and, even if the windmill runs easily enough, its output is so small that it is of no particular use. At other times the wind is so strong as to make it difficult to protect the windmill from injury. It is hardly possible to so construct a windmill that it can utilize both very weak and very strong winds. Constructors are therefore generally restricted to the most common wind velocities of 3-10 m (10-33 ft.) per second inland and somewhat higher on the coast. Special regulating devices must be provided in order to let the wind pass unobstructed when it is very strong, so that it will do no damage. The irregularity of the wind also has the disadvantage that the energy produced varies greatly at different times and is generally not adapted to the task. In general, it cannot be assumed that the wind will have just the desired force at a given time. At other times, when the wind is just right, there will be no suitable use for the energy. It is therefore necessary, either to store the energy or to provide suitable uses for it, which will be independent of the time when the energy is available (for example, irrigation pumps). The storing of the energy (e.g., in the form of electricity in storage batteries) considerably increases its cost.

Another difficulty is the low revolution speed of the windmills. For reasons which will be discussed farther on, the peripheral velocity of the vane tips must be able to stand, within certain limits, in a variable ratio to the wind velocity. Since the diameter of a windmill is generally quite large, in order to develop sufficient power, its revolution speed is usually very small in comparison with most other machines. Hence, in order to operate another machine with the windmill, a multiplying gear must generally be used, which likewise increases the cost of the energy. The difficulty of operation increases with the diameter of the windmill, since the revolution speed is thereby diminished, while the power to be transmitted is increased. It is therefore desirable to obtain as high a revolution speed of the windmill as possible, i.e., to work with the highest possible ratio of the peripheral velocity to the wind velocity. Unfortunately, such high-speed windmills have other serious disadvantages which limit their speed.

In what follows, we will consider two of these problems more closely. The first one may be expressed in the question "How can energy be obtained from the wind and what power can be developed with a windmill of a given size at a given wind velocity?" We will also consider the problems connected with the speed.

II. How the Energy is Obtained

We will first consider how the air can be made to give up its kinetic energy. We can imagine some such arrangement as shown in Figure 1. A flat plate of area F is placed facing the wind. The wind, blowing against this plate with a velocity v, exerts on it a force W, which is proportional to the area F, the square of the velocity v and the air density ρ. For air under normal conditions, $\rho = 1/8$ kg sec² m⁻⁴. This relation is usually expressed as follows:

$$W = c_W \frac{\rho}{2} F v^2 \tag{1}$$

The factor c_W is termed the coefficient of drag and depends on the shape of the object. If the object is allowed to move in the direction of this resisting force, it performs work which can be utilized (for example) to raise a weight. Since, however, due to the motion of the object, the relative velocity of the wind is diminished, the resistance or drag of the object itself is also diminished. If the velocity of the object is v', the relative velocity of the air is $v - v'$ and

$$W = c_W \frac{\rho}{2} F(v - v')^2 \tag{2}$$

The work done per second is

$$L = W v' = c_W \frac{\rho}{2} F(v - v')^2 v' \tag{3}$$

This work is the greatest for a given area F and wind velocity v, when $v' = \frac{1}{3} v$, namely

$$L_{max} = \frac{4}{27} c_W \frac{\rho}{2} F v^3 \tag{4}$$

In this process, the wind itself does the work

$$L' = W v \tag{5}$$

per second, since the air moves with the velocity v and exerts a force W in the direction of motion. For $v' = v/3$, the wind therefore does three times as much work as that obtained by the above arrangement. Two-thirds of the power developed by the wind is converted into vortices (and finally, into heat) and only 1/3 is saved. If, as was explained at the beginning, this low efficiency is not decisive as regards the suitability of the arrangement, it must nevertheless give cause for apprehension. It may be assumed that any arrangement which works with less loss, even with the same area, will furnish more power for the same cost.

In the above-described arrangement, the force W exerted by the air on the object involved a loss of energy, in that the work $W(v - v')$ was converted into vortical energy. This leads us to inquire as to whether work is conceivable without this loss of energy. This is actually the case. If a suitably shaped object is chosen, the ~~power~~ force P does not lie in the direction of the relative air flow, but forms an acute angle with it (Fig. 2). If this force is resolved into a component W in the direction of the air flow and a component A perpendicular to it, then only the component W, the real resistance or "drag," determines the amount of energy consumed in the production of vortices. The energy lost per second (L_{lost}) is W_c, in which c is the wind velocity relative to the object. The other component, the "lift," causes no loss of energy, since it is perpendicular to the direction of motion. This lifting force is therefore similar to a centripetal force, since it only deflects the air from its course without affecting its energy. In order to obtain great efficiency with only a small loss of energy, objects must be used of such a shape that they will yield the greatest possible lift and the smallest possible drag. Such objects are called "wings" or "supporting wings." The drag-lift ratio, which indicates the quality of a wing is called the "glide coefficient," because it indicates the minimum slope of the flight path at which the wing can make a stable gliding flight. We will express the coefficient of glide by ϵ. It is therefore

$$\epsilon = \frac{W}{A} \tag{6}$$

Corresponding to the resistance of a simple object, we can express the lift and drag of a wing as follows:

$$A = c_a \frac{\rho}{2} F c^2 \tag{7}$$

$$W = c_w \frac{\rho}{2} F c^2 \tag{8}$$

in which c_a and c_w are proportionality factors, which are affected only by the shape and position of the wing. The maximum projected area of the wing is generally chosen as the reference area F. For a rectangular wing with span l and chord t, we would therefore have $F = l\,t$.

We will now use such a wing for obtaining energy from the wind. Let v represent the velocity of the wind. The wing is moving perpendicularly to the wind with the velocity u (Fig. 3). The air then flows relatively to the wing with a velocity c, which, from the composition of the components v and u (Fig. 4) yields

$$c = \sqrt{v^2 + u^2} \tag{9}$$

The direction of the relative velocity c forms with the wind direction v the angle β (Figs. 3-4), for which we have the relation

$$\tan\beta = \frac{u}{v} \tag{10}$$

Since the lift A, which the wing acquires in this motion, is perpendicular to the relative velocity c, it likewise forms the angle β with the direction of motion u of the wing (Figs. 3 and 5). The effective power depends on the component $T_1 = A \cos\beta$, of the lift A in the direction u. The motion is opposed by the drag component $T_2 = W \sin\beta$. In the direction of the velocity u, there is accordingly exerted the total force

$$\left.\begin{aligned} T = T_1 - T_2 &= A \cos\beta - W \sin\beta \\ &= A \cos\beta\,(1 - \epsilon \tan\beta) \end{aligned}\right\} \tag{11}$$

in which $\epsilon = W/A$, the glide coefficient of the wing. The effective power is accordingly

$$L_N = T\,u = Au\cos\beta\,(1 - \epsilon\tan\beta) \tag{12}$$

The power components falling in the wind direction v yield the thrust

$$\left.\begin{aligned} S = S_1 + S_2 &= A\sin\beta + W\cos\beta \\ &= A\sin\beta\,(1 + \epsilon\cot\beta) \end{aligned}\right\} \tag{13}$$

The energy supplied by the wind is

$$L_W = S\,v = A\,v\sin\beta\,(1 + \epsilon\cot\beta) \tag{14}$$

If we consider that $v\sin\beta = u\cos\beta$, we then have

$$L_N = L_W\frac{1 - \epsilon\tan\beta}{1 + \epsilon\cot\beta} = L_W\frac{1 - \epsilon\frac{u}{v}}{1 + \epsilon\frac{v}{u}} \tag{15}$$

If the wing drag and consequently the glide coefficient were zero, the energy L_N thus obtained would equal the energy L_W furnished by the wind, i.e., we would have an energy conversion without loss. In reality there is always some resistance or drag and consequently some loss. The effective energy in the ratio

$$\eta_1 = \frac{1 - \epsilon\frac{u}{v}}{1 + \epsilon\frac{v}{u}} \tag{16}$$

is less than the energy furnished by the wind. This ratio η_1 can therefore be regarded as the efficiency of the wing. Since the glide coefficient ϵ is generally a small number (0.02 to 0.1 according to the fineness of the wing) the efficiency will be poor only when the ratio u/v of the wing velocity to the wind velocity is very small with reference to unity. In the former case $\epsilon \frac{u}{v}$ (in the numerator) and in the latter case $\epsilon \frac{v}{u}$ (in the denominator) are of such an order of magnitude that the efficiency suffers appreciably. The former case $\frac{u}{v} \gg 1$ is the more important one, because the ratio $\frac{u}{v}$ is limited upward through consideration for the efficiency. We will come back to this later.

We will now determine how large the wing area must be in order to obtain a given power. According to equation (7) the lift is

$$A = c_a \frac{\rho}{2} F c^2$$

in which c_a denotes the lift coefficient, which is of the order of magnitude of 1; ρ the air density; F the wing area and

$$c = \sqrt{v^2 + u^2} = u\sqrt{1 + \left(\frac{v}{u}\right)^2}$$

the velocity of the air with reference to the wing (Figs. 3 and 4). If we put this value in equation (12) and note that $\cos\beta = \frac{v}{c}$ and $\tan\beta = \frac{u}{v}$, we thus obtain the effective energy

$$\left.\begin{aligned} L_N &= c_a \frac{\rho}{2} F c^2 u \frac{v}{c} \left(1 - \epsilon \frac{u}{v}\right) \\ &= c_a \frac{\rho}{2} F v^3 \left(\frac{u}{v}\right)^2 \sqrt{1 + \left(\frac{v}{u}\right)^2} \left(1 - \epsilon \frac{u}{v}\right) \end{aligned}\right\} \qquad (17)$$

If we compare this result with equation (4) and consider that the c_w in that equation and the c_a in equation (17) are both of the order of 1, it is obvious that we now obtain a considerably greater power with the same area F, the same wind velocity v, and the same air density ρ. On the one hand, we avoid the factor $\frac{4}{27}$ in equation (4), beyond which we still further increase the power by making $\frac{u}{v}$ large. If, for example, $\frac{u}{v} = 3$, the factor increasing the power becomes

$$\left(\frac{u}{v}\right)^2 \sqrt{1 + \left(\frac{v}{u}\right)^2} = 3^2 \times 1.05 = 9.45.$$

The last factor $\left(1 - \epsilon \frac{u}{v}\right)$ brings an impairment which plays no great role, however, so long as the glide coefficient of the wing is small and $\frac{u}{v}$ is not excessive. In our example, with $\frac{u}{v} = 3$ and $\epsilon = 0.05$, it would be

$$\left(1 - \epsilon \frac{u}{v}\right) = (1 - 0.05 \times 3) = 0.85.$$

On the assumption that c_a in equation (17) and c_w in equation (4) are just alike, we would accordingly obtain a power which is multiplied by the factor $\frac{27}{4} \times 9.45 \times 0.85$ and therefore about 54 times as great as with the former arrangement. For a velocity ratio $\frac{u}{v} = 2$, this factor would be about 27 and for $\frac{u}{v} = 1$, it would be about 9. This great difference would justify the unconditional choice of the second arrangement, namely, the utilization of the wing lift instead of the drag, even though the construction of the wing should be somewhat more expensive than that of a simple resisting body.

III. The Windmill

We must now consider the question as to how the motion of the vanes for the production of energy is practically accomplished. Thus far we have considered a wing or airfoil moving in a straight line. This method of energy production corresponds to the use of the wind by sailing vessels.* With stationary power plants, like windmills, the vanes or blades must move in closed circuits, the simplest way being to let them rotate in a suitable manner about a shaft. There has been no lack of attempts to use resisting bodies for the transfer of energy. They have all proved uneconomical, however, which is not strange in view of the above explanation. We will here confine ourselves, therefore, to the customary way of using vanes.

The usual method is to fasten a number of vanes to a shaft (Fig. 6), to place this shaft in the direction of the wind and to let the whole system, i.e., the "windmill" revolve about the axis of the shaft. If w is the angular velocity of the windmill, any point on a vane at the distance r from the axis has the velocity $u = r\omega$ perpendicular to the wind. It might be imagined, on the basis of the considerations in the preceding chapter, that the forces acting on the vanes and the effective

*The glide coefficient is here rather poor since, for the sake of stability, the vanes cannot be placed high enough for a good coefficient (Induced Drag: See Betz, "Einführung in die Theorie der Tragflugel," Die Naturwissenschaften, 1918, p.557).

power could be calculated directly. That the vane velocity varies from point to point, would only increase the difficulty of the calculation, but would constitute no very great obstacle. That something has been overlooked, however, is obvious from the fact that, according to the previous considerations, the power is proportional to the wing area. We would accordingly be able to obtain any desired amount of energy with a windmill of quite small diameter by simply widening the vanes or increasing their number, which is contrary to all experience. Obviously, the vanes disturb one another, so that their effect in the windmill differs from what it would be if they were exposed separately to the wind. We must therefore first consider this mutual interference.

In the preceding chapter, we learned to know the forces which the air exerts on a wing. The wing exerts the same forces on the air, only with the opposite sign. In particular, the component S in the wind direction reduces the wind velocity, while the thereto vertical component T only deflects the air flow. When the vanes move one behind another, each one works in a zone where the air flow has already been more or less disturbed by the preceding vanes.* An accurate analysis of these

*Even with only one vane, the motion of the air in the vicinity of the vane is disturbed by the vane itself. The effect of this disturbance is already included in the experimentally found values of c_a and c_w, because this disturbance is present in the determination of these values. It must be considered, however, in using a differently shaped vane from the one used in the determination (Induced drag. See the last preceding footnote).

disturbing influences would be very troublesome. For our purpose, however, the task may be simplified by asking whether we can estimate the mutual interference of the vanes, under simple but surely too favorable assumptions, so as to obtain a maximum value of the power to be obtained. The actual power of the windmill would then lie more or less below this maximum value, according to the suitability of its design. The following consideration will serve to answer this question.

The available energy in the wind is present in the form of kinetic energy. If v is the velocity of the air, then a portion of the air with the mass m has the kinetic energy $\frac{m}{2} v^2$. If, as heretofore, ρ represents the air density, that is, the mass of a unit volume, the energy is $\frac{\rho}{2} v^2$. If we now extract energy from the air by means of a windmill, the kinetic energy behind the windmill (hence also the velocity v) must be less than in front of it. If we designate the velocity in front of the windmill by v_1 and behind the windmill by v_2 (Fig. 7,a), then $v_2 < v_1$. We know also that the vanes exert a force S on the air opposite to the direction of motion of the wind, thus causing this reduction in the velocity of the wind. The transition from the higher to the lower velocity does not occur suddenly, since the slower air requires a larger cross section than the faster air and the lines of flow require time for spreading out. The process is such that even in front of the windmill the air is somewhat retarded. Its velocity is thereby converted into pressure. Consequently, the air strikes the windmill with diminished velocity but with increased pressure. The energy extraction by the windmill first causes only a diminution of the pressure energy, so that the air which entered the windmill with increased pressure, leaves it with diminished pressure. The velocity itself cannot change in the thickness of the windmill. Only behind the windmill the velocity diminution again continues until the normal air pressure is restored (Fig. 7,b).

Since the vanes do not uniformly fill the windmill area, but often leave quite large intervening spaces, it might be thought that the air would be retarded only just where the vanes are, thus leaving the air between the vanes to flow through quite unimpeded. Such is not the case, however. Since the vanes revolve, they affect (though not simultaneously, still at very short intervals) all the air which flows through the circular area traversed by the vanes (annular surfaces when the vanes do not extend clear to the axis). Of course there is some lack of uniformity in this periodical effect, but accurate calculations show that it is very small under actually occurring conditions. For the present investigation, we can therefore imagine the windmill replaced by a perforated disk, which exerts a retarding effect on the air and thereby extracts energy from it. The tangential forces T and their effect (deflection of the flow direction) will be here disregarded. We can indeed imagine that a stationary rectifying device is installed behind the windmill, to restore the air current to its original direction.

If the air mass m flows through the windmill per second and its velocity is reduced from v_1 to v_2, there is extracted from the air per second the energy

$$L = \frac{m}{2} (v_1^2 - v_2^2). \tag{18}$$

This much energy is available, but only a portion of it is actually obtained, due to the losses involved, especially in the above-mentioned wing efficiency. It will naturally be endeavored to extract the most possible energy from the wind, in order to be able actually to obtain the greatest possible amount of energy. We must, therefore, consider how great the energy L can be in equation (18) in the most favorable case with a given windmill diameter D, area $F_0 = \frac{D^2 \pi}{4}$ and wind velocity v_1. A cursory consideration of equation (18) might lead one to think the power L would be a maximum when $v_2 = 0$, that is, when all the energy is extracted from the passing air. Such is not the case, however, since with decreasing v_2, the mass m of air flowing through the windmill per second decreases. The more the air is retarded the less air flows through the windmill. A portion of the air flows around the obstruction without giving up its energy. In order to determine the air mass m flowing through per second, we must know its velocity v'. This mass is given by the formula

$$m = \rho \, F_0 \, v'.$$

The velocity v' evidently lies between the velocities v_1 and v_2 before and behind the windmill. It can be shown that v' is exactly the arithmetical mean of v_1 and v_2:

$$v' = \frac{v_1 + v_2}{2} .$$

For this purpose the following consideration will help. If the mass m flows through the windmill per second and its velocity is thereby reduced from v_1 to v_2, it then extracts from the air, as already established, the energy

$$L = \frac{m}{2} (v_1^2 - v_2^2)$$

per second. On the other hand, when the air flows through the windmill with the velocity v' and thereby experiences a resistance S, its yield of energy per second is

$$L = Sv' \tag{19}$$

It must therefore be

$$\frac{m}{2} (v_1^2 - v_2^2) = Sv' .$$

The resistance S must, however, reduce the velocity of the mass m from v_1 to v_2. Hence, according to the law of momentum

$$S = m (v_1 - v_2) \tag{20}$$

If we substitute this value in the preceding equation and consider, moreover, that $v_1^2 - v_2^2 = (v_1 - v_2)(v_1 + v_2)$, we obtain

$$\frac{m}{2} (v_1 - v_2)(v_1 + v_2) = m (v_1 - v_2) v'$$

or

$$\frac{v_1 + v_2}{2} = v' . \tag{21}$$

We now have everything necessary for determining the energy L for any initial and final velocities v_1 and v_2. We accordingly have

$$m = \rho\, F_0\, v' = \rho\, F_0\, \frac{v_1 + v_2}{2} \tag{22}$$

and

$$\left.\begin{aligned} L &= \frac{m}{2}\,(v_1^2 - v_2^2) = \frac{\rho\, F_0}{4}\,(v_1 + v_2)\,(v_1^2 - v_2^2) \\ &= \frac{\rho\, F_0\, v_1^3}{4}\left(1 + \frac{v_2}{v_1}\right)\left[1 - \left(\frac{v_2}{v_1}\right)^2\right] \end{aligned}\right\} \tag{23}$$

The energy L will accordingly be the greatest when we see to it that

$$\frac{v_2}{v_1} = \frac{1}{3} \tag{24}$$

We then have

$$L_{max} = \frac{16}{27}\,\frac{\rho}{2}\,F_0\,v^3 \tag{25}$$

Since it is hardly possible, with an actual windmill, to obtain this favorable ratio $\frac{v_2}{v_1} = \frac{1}{3}$ for the whole amount of air flowing through and since, moreover, as we already know, losses occur on the vanes themselves, the actual output of the windmill is less than the theoretical maximum value. The ratio of the actual energy output L to the theoretical output L_{max} furnishes a criterion for the quality of the construction and corresponds to the efficiency in other machines. We will designate this ratio

$$\zeta = \frac{L}{L_{max}} \tag{26}$$

as the efficiency. For representing the experimental results, a somewhat different value, the efficiency ratio

$$c_l = \frac{L}{\frac{\rho}{2} F\, v^3} = \frac{16}{27}\, \zeta \qquad (27)$$

is customary (Fig. 9). Thereby it must be remembered that the theoretical limit of c_l is not 1 but $\frac{16}{27}$.

IV. Vane Dimensions and Rotational Speed

We have now learned the limits of the output ability of a windmill and the preliminary assumption for the maximum power output. The next question concerns what is to be done in order to effect the most favorable retardation $\frac{v_2}{v_1} = \frac{1}{3}$ for the maximum output. Obviously, we here have a condition for the size and number of the vanes, since, if a too dense system of vanes is presented to the wind, the latter is retarded too much; and, if the system is not dense enough, too much energy passes unused through the windmill. It is not only the area of the vanes which is responsible for this, but also the speed at which they revolve. In chapter II the energy absorbed by a single vane was calculated (equation (17)) and it was found to be affected not only by the vane area but also very greatly by the ratio u/v, that is, by its own motion. This shows that, in order to obtain the same result, the vane area must be reduced in proportion to the increase in the revolution speed of the vanes.

Equation (17) for a single vane cannot be transferred directly to the vanes of a windmill, since the vanes mutually affect one another. We now know, however, that this is due to the fact that the wind velocity which acts on the vanes is not the undisturbed velocity v, but the smaller velocity v' and that the output is correspondingly reduced. We can apply equation (17) to the vanes of a windmill by substituting the velocity v' for the undisturbed velocity v. From what has preceded, we know that we must try to make $v_2 = \frac{v}{3}$ and $v' = \frac{v + v_2}{2} = \frac{2v}{3}$.

All the preliminary conditions have now been stated for calculating the requisite vane area. A slight difficulty, however, resides in the fact that the velocity u of the windmill vane varies with the radius. If ω is the angular velocity of the windmill, then $u = r\,\omega$. We must therefore consider each radius separately. If, instead of the whole windmill area, we consider an annular area of the width b and the mean radius r (Fig. 8), we find the latter to be $\Delta F = 2\,r\,\pi\,b$. For the air mass $m = 2\,r\,\pi\,\rho\,b\,v'$ flowing through the windmill per second, this area must exert an axial force of

$$\Delta S = m\,(v_1 - v_2) = m\,\frac{2}{3}\,v = 2\,r\,\pi\,b\,\rho\,\frac{4}{9}\,v^2 \text{*}$$

*This is not quite accurate since, during the equalization flow before and behind the windmill, the air particles also exert slight axial forces on one another (Cf D. Thoma, "Grundsätzliches zur einfachen Strahltheorie der Schraube," Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1925, p. 206.

If the windmill has n vanes of the width t, and if the angular velocity of the windmill is ω, then, for the annular area under consideration, $u = r\omega$ and $c = r\omega\sqrt{1 + \left(\frac{v'}{r\omega}\right)^2}$. According to equation (17) we obtain, after a slight transformation, while remembering that $\Delta L = \Delta S\ v'$,

$$\Delta S = c_a \frac{\rho}{2}\, n t b v'^2 \left(\frac{r\omega}{v'}\right)^2 \sqrt{1 + \left(\frac{v'}{r\omega}\right)^2}\left(1 - \epsilon\,\frac{r\omega}{v'}\right)$$

$$= c_a \frac{\rho}{2}\, n t b v^2 \left(\frac{r\omega}{v}\right)^2 \sqrt{1 + \frac{4}{9}\left(\frac{v}{r\omega}\right)^2}\left(1 - \frac{3}{2}\,\epsilon\,\frac{r\omega}{v}\right)$$

By comparison with the above equation, we obtain

$$c_a \frac{nt}{2\,r\,\pi} = \frac{16}{9}\left(\frac{v}{r\omega}\right)^2 \frac{1}{\sqrt{1 + \frac{4}{9}\left(\frac{v}{r\omega}\right)^2}\left(1 - \frac{3}{2}\,\epsilon\left(\frac{r\omega}{v}\right)\right)} \qquad (28)$$

The expression $\frac{n\,t}{2\,r\,\pi}$ represents the ratio of the vane width t to the interval $\frac{2\,r\,\pi}{n}$ between vanes, that is, the "vane density," if we may use this expression. The second terms of these ratios do not generally differ much from unity, so that $\left(\frac{v}{r\,\omega}\right)^2$ appears as the essential factor, i.e., the square of the ratio of the wind velocity to the vane velocity with the given radius. In fact, the "vane density" is inversely proportional to the vane velocity. The choice, within certain limits, of the lift coefficient c_a renders it possible to adapt the vane width to the desired structural conditions. With a constant c_a, the vane width, for example, would have to be greater toward the hub. Generally, however, it is more convenient to leave the vane width practically constant and allow the lift coefficient to diminish toward the circumference of the windmill. Very near the axis, the dimensions according to equation (28) are no longer possible, since c_a cannot be increased beyond a certain degree and there is no longer space enough for the resulting vane width. Moreover, the tangential deflection velocities, which are disregarded in our discussion, have a noticeably unfavorable effect. Ordinarily, the central portion of the windmill is not utilized, since it could operate only under unfavorable conditions.

Since the vane velocity $r\,\omega$, for any radius r, is proportional to the velocity $u_o = R\,\omega$ of the vane tips $(R = \frac{D}{2} =$ the radius of the windmill), the "vane density" is essentially determined by the ratio $\left(\frac{u_o}{v} = \frac{R\,\omega}{v}\right)$ of the peripheral velocity of the windmill to the velocity of the wind itself. This ratio expresses the rotational speed of the windmill. The faster a windmill is designed to rotate, the smaller must be its "vane density."

A high rotational speed is desirable for two reasons. In the first place, a smaller vane area is required and the windmill is correspondingly simpler. In the second place, as was mentioned in the introduction, the transmission of the energy to the driven machines is facilitated. Most machines require a considerably greater rotational speed than that of the windmill, so that a multiplying gear is almost always needed. The transmission is much simplified by increasing the rotational speed of the windmill. Any great increase in the rotational speed is precluded, however, by structural difficulties.

<u>1. The glide coefficient</u>.- The factor $\left(1 - \epsilon \frac{r\,\omega}{v'}\right)$, which reduces the output, has a decided effect as soon as $\frac{r\,\omega}{v'}$ reaches the order of magnitude $\frac{1}{\epsilon}$. For high-speed windmills, therefore, the vane profiles must be very carefully constructed and must have a sufficiently low glide coefficient $\left(\frac{W}{A}\right.$ ratio). This will naturally increase the cost of the windmill and offaet the advantage of greater simplicity.

<u>2. Starting moment</u>.- With a high-speed windmill, the requisite starting force is obtained through the great intrinsic speed of the vanes. This effect is produced, however, only when the vanes really have great intrinsic speed. While the windmill is standing still, the force exerted on the small vanes of a high-speed windmill is less than that exerted on the large vanes of a low-speed windmill. Consequently, with like frictional conditions, a low-speed windmill will be started by a weaker wind than a high-speed windmill. Since a windmill must often stop and start again, due to the irregularity of the wind, the more difficultly started high-speed windmill is often unable to utilize weak winds which can be utilized, however, by low-speed windmills.

3. The centrifugal forces which a windmill, in revolving, exerts on its support, due to errors in the center of gravity, are naturally greater at high speeds than at low speeds. The supporting tower and its foundations must therefore be made stronger, or the windmill itself must be better balanced. Both, however, mean increased cost of construction.

It is probably due to these difficulties that very high-speed windmills have not yet been adopted. There has, however, been a gradual development toward higher speed. The following values of $\frac{u_o}{v}$ are common:

$\frac{u_o}{v}$ = 1 to 2, low-speed windmills with many vanes (wind turbines);

$\frac{u_o}{v}$ = 2 to 3, transition to high-speed windmills (the old four-vane windmills);

$\frac{u_o}{v}$ = 3 to 4, high-speed windmills;

$\frac{u_o}{v}$ = 4 : Probably none is in use, though a few have been made.

V. Experimental Results

In the preceding chapters we have considered the process of energy production and its limits. We are now prepared, when we know the energy output of an actual windmill, to determine its efficiency. Unfortunately, very few reliable data are available concerning the actual performances of windmills.

There are two methods for determining their power:

1. By investigating windmills made for practical purposes;
2. By testing models in an artificial air stream.

Both methods have great disadvantages. By the first method, the determination of the effective output with the usual degree of accuracy in technical tests encounters no great difficulty. On the contrary, the determination of the wind velocity, on which the attainable effective output essentially depends, encounters great difficulties. The possibilities of error are very great in the measurement of wind velocities, and considerable technical knowledge and experience are necessary, in order to obtain fairly correct values. Since the energy output is proportional, however, to the third power of the wind velocity (equation (25)), any error in measuring the wind velocity has a very noticeable effect. If, for example, the wind velocity is measured only 10% too low, the corresponding efficiency ratio is about 37% too high. If, however, the velocity is really twice as great as measured, which is entirely possible with unskillful use of the instruments, the efficiency ratio will appear to be eight times as great as it really is. The measurement is rendered still more difficult by the irregularity of the wind. Most instruments give the approximate mean value of the velocity, while for the windmill output, the third power of the velocity is decisive. If, for example, the wind has velocities of four meters and eight meters per second alternatingly for equal periods, the measuring instrument would indicate a mean of about six meters per second. From this we would calculate $v_m^3 = 6^3 = 216\ m^3/sec^3$. In reality, however, the decisive result for the output would be $v'^3_m = \frac{1}{2}(4^3 + 8^3) = 288\ m^3/sec.^3$. Since the manufacturers have an interest in making their windmills appear as efficient as possible, they naturally use the best of the various test results in their advertising, and it thus happens that much too high performances are announced in very many catalogs. Relatively good results have recently been obtained in the Agricultural Institute of Oxford University.*

The second method of windmill investigation, namely, experimentation with models, avoids the great difficulties offered by the natural wind by utilizing an artificially produced air stream, whose velocity is much more conveniently and accurately determined. We must, however, accept the inaccuracies resulting from the use of a relatively small model instead of an actual windmill. It is practically impossible to imitate accurately a real windmill with all its details, such as screws, rivet heads, etc. Even if this were possible, the forces measured on the model would not correspond exactly to the ones on the full-scale windmill, since, due to the effect of the viscosity of the air, the law of similitude for transferring the results from the model to the full-scale windmill is not perfectly fulfilled, especially

*"A Report on the Use of Windmills for the Generation of Electricity." Clarendon Press, Oxford, 1926.

for small parts. The errors, however, are generally small if, in making the model, these difficulties are avoided by omitting the parts which are difficult to copy. Figure 9 gives some results obtained with windmill models at the Göttingen Aerodynamic Institute,* for a very low-speed windmill (a), an extremely high-speed windmill (c), and a modern medium type (b).

The abscissa represents the ratio $\frac{u_o}{v}$ of the peripheral velocity to the wind velocity, against which the above-mentioned efficiency ratio

$$c_l = \frac{L}{\frac{\rho}{2} v^3 \frac{D^2 \pi}{4}}$$

is plotted. As regards this efficiency ratio, we know from what has preceded, that its theoretical limit is $\frac{16}{27}$. In order better to take into account the starting conditions, we have plotted, in addition to the efficiency ratio c_l, also the torque ratio

$$c_d = \frac{M}{\frac{\rho}{2} v^2 \frac{D^2 \pi}{4} \frac{D}{2}} = c_l \frac{v}{u_o}$$

$\left(M = \text{torque of windmill} = \frac{L}{\omega}\right)$. It is seen that the low-speed windmill attains its maximum efficiency at about $\frac{u_o}{v} = 0.9$; the medium one at about $\frac{u_o}{v} = 1.5$; and the high-speed one at about $\frac{u_o}{v} = 3$. It is also seen that the starting torque (for $\frac{u_o}{v} = 0$) of the high-speed windmill is considerably smaller than that of the low-speed windmill.

*"Ergebnisse der Aerodynamischen Versuchsanstalt zu Göttingen," Report III, R. Oldenbourg, Muncih, 1927.

Translation by Dwight M. Miner,
National Advisory Committee for Aeronautics.

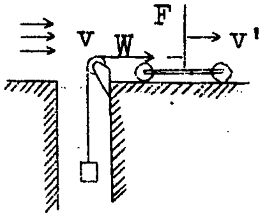

Fig.1

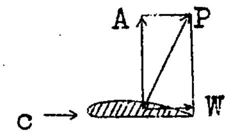

Fig.2 Forces acting on a wing.

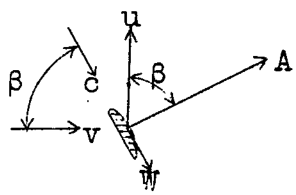

Fig.3

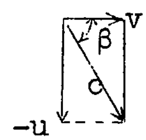

Fig.4

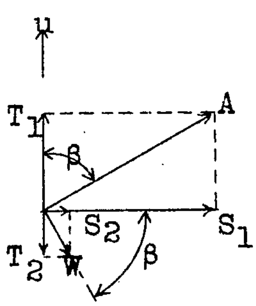

Fig.5

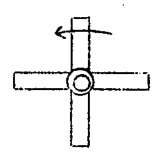

Fig.6 Windmill.

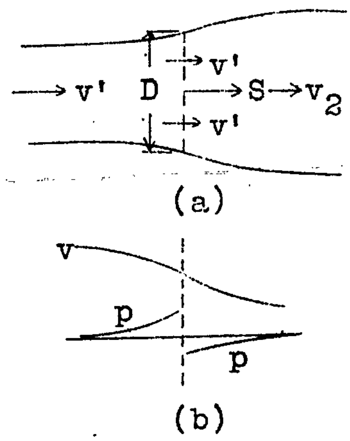

Fig.7(a).Flow through a windmill.
(b).Course of velocity v and of pressure p of the air before and behind windmill.
(Windmill is indicated by dash lines.)

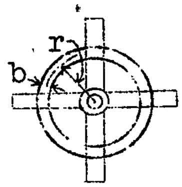

Fig.8

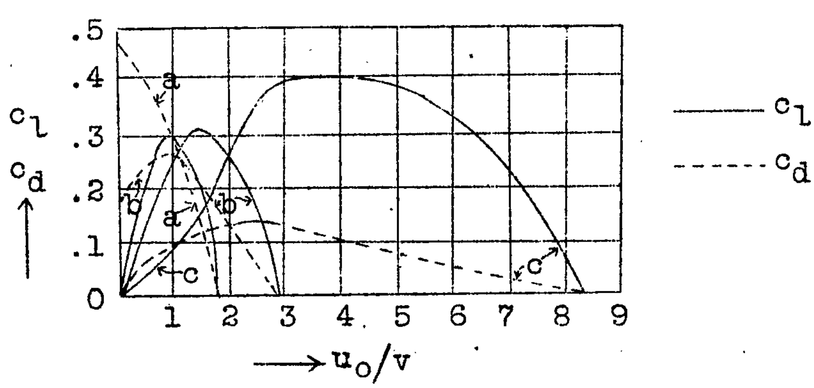

Fig.9

a, Very low-speed windmill.
b, Modern medium type windmill.
c, Extremely high-speed windmill.

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